9a) $f(x) = \frac{x}{x^2+2}$

9a
11a

$$f'(x) = \frac{(1)(x^2+2) - x(2x)}{(x^2+2)^2} = \frac{-x^2+2}{(x^2+2)^2}$$

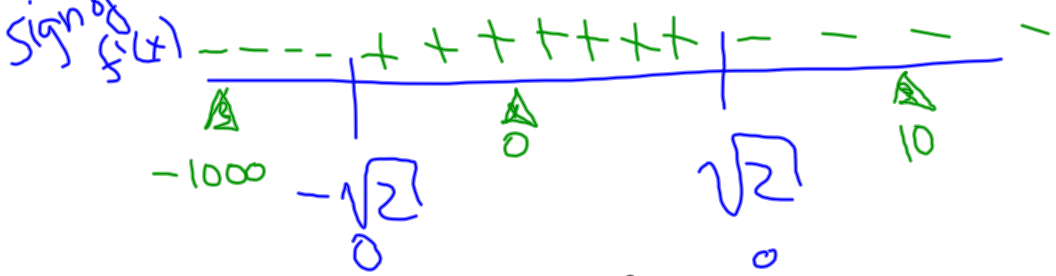
$f'(x)$ undefined: $(x^2+2)^2 = 0$?

nowhere

$f'(x) = 0 \Rightarrow -x^2+2 = 0 \Rightarrow x = \pm\sqrt{2}$

$\frac{2-x^2}{(x^2+2)^2}$

sign of $f'(x)$



critical pts: @ $x = -\sqrt{2}, +\sqrt{2}$

relative minimum: @ $x = -\sqrt{2}$

rel maximum: @ $x = +\sqrt{2}$

11a) $f(x) = x^{1/3}(x+4) = x^{4/3} + 4x^{1/3}$

$f'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3}x^{-2/3}$

$= \frac{4}{3}\left(x^{1/3} + x^{-2/3}\right) = \frac{4}{3x^{2/3}}\left(x+1\right)$

(7a)

$f(x) = x^3 + 3x^2 - 9x + 1$

$f'(x) = 3x^2 + 6x - 9$

$3x^2 + 6x - 9 = 0$

$3(x^2 + 2x - 3) = 0$

$3(x+3)(x-1) = 0$

$x = -3$ or $x = 1$ — crit (stationary)

NO undefined pts.

increasing: $(-\infty, -3] \cup [1, \infty)$

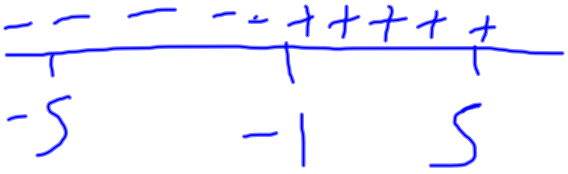
decreasing: $[-3, 1]$

$f'(x) = 3x^2 + 6x - 9$

$f''(x) = 6x + 6$

$6(x+1) = 0$

crit (inflection) $x = -1$



C-dn: $(-\infty, -1]$

C-up: $[-1, \infty)$

11a

7a

12

7b) $f(x) = x^4 - 6x^2 - 3$

$f'(x) = 4x^3 - 12x = 4(x^3 - 3x) = 4x(x^2 - 3)$ $\sqrt{x^2 = 3}$

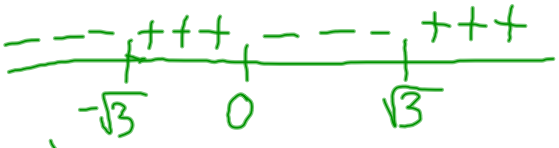
f' undefined → NOPE

crit pts

$x = 0$

$x = \pm\sqrt{3}$

rel min is -12 at $x = -\sqrt{3}$



$f'' = 12x^2 - 12 = 12(x^2 - 1)$

$f'' = 0$ $\sqrt{x^2 = 1}$

f'' undefined → NEVER $x = \pm 1$

c-up: $(-\infty, -1) \cup (1, \infty)$

c-dn: $(-1, 1)$

12) $f(x) = x^{4/3} - 6x^{1/3}$

$f'(x) = \frac{4}{3}x^{\frac{1}{3}} - \frac{2x^{\frac{2}{3}}}{x^{-2/3}}$ $\quad x^{-2/3}\left(\frac{4}{3}x - 2\right)$

Critical pt.

$\frac{3}{2}$ Undefined $x = 0$

$x^{-2/3} = 0$

$\frac{1}{x^{2/3}} = 0$ ✗

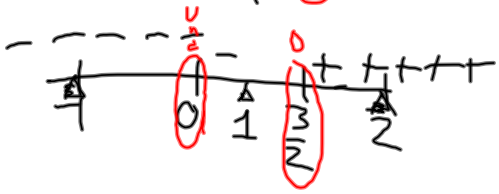
$\frac{4}{3}x - 2 = 0 \quad x = \frac{3}{2}$

$+2 \quad +2$

$\frac{4}{3}x = 2$

$\frac{4}{4} \quad \frac{4}{3}$

$\frac{2}{\frac{4}{3}} \cdot \frac{3}{2}$



$f'(x) = x^{-2/3}\left(\frac{4}{3}x - 2\right)$

always pos

$(-1)^{2/3} = \sqrt[3]{(-1)^2}$

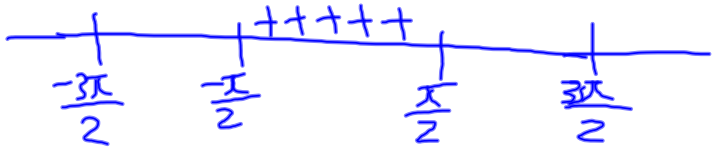
$(-1)^{-2/3}\left(\frac{4}{3}(-1) - 2\right) = \frac{1}{(-1)^{2/3}}\left(-\frac{4}{3} - 2\right)$

$f(x) = |\sin x| = \begin{cases} \sin x, & x \in [0, \pi] \\ -\sin x, & x \in [-\pi, 0] \end{cases}$

$f' = \cos x$

$-\cos x$

Cpts $= \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$



24) $f(x) = \frac{1}{2}x - \sin x$, $0 < x < 2\pi$

$f'(x) = \frac{1}{2} - \cos x$

critical points: $f'(x) = 0$ or $f'(x)$ undefined

$\frac{1}{2} - \cos x = 0$

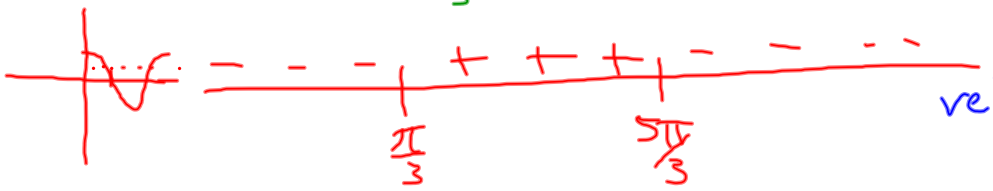
$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$f'(x)$ undefined: none

inc: $\left[\frac{\pi}{3}, \frac{5\pi}{3}\right]$

dec: $\left(0, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right)$



rel min: @ $x = \frac{\pi}{3}$

rel max: @ $x = \frac{5\pi}{3}$

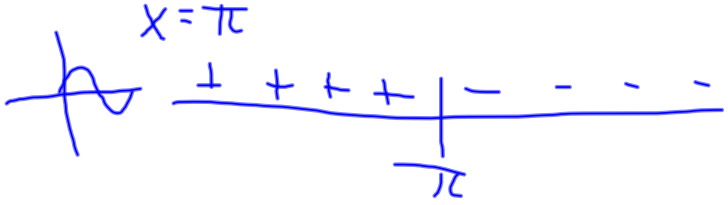
$f''(x) = \sin x$

possible pts of inflection: $f'' = 0$ or f'' undefined

$\sin x = 0$

$x = \pi$

none



c-up: $(0, \pi)$

c-dn: $(\pi, 2\pi)$

pt of inflection @ $x = \pi$

y-int: $y = 0$

x-int: ?

5.2/10

$f(x) = \frac{x^2-3}{x^2+1}$

$f'(x) = \frac{(x^2-3)'(x^2+1) - (x^2-3)(x^2+1)'}{(x^2+1)^2}$

$\frac{2x(x^2+1) - 2x(x^2-3)}{(x^2+1)^2}$

$\frac{2x[(x^2+1)-(x^2-3)]}{(x^2+1)^2}$

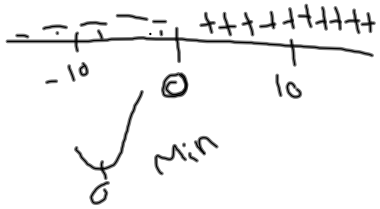
$\frac{2x[4]}{(x^2+1)^2} = \frac{8x}{(x^2+1)^2}$

$f(x)=0$ ← Critical

$f'(x)$ undefined ($x^2+1 \neq 0$ where)

$8x=0$ ← Stationary

$x=0$



$\frac{8x}{(x^2+1)} \quad \frac{(8x)'(x^2+1) - (x^2+1)'(8x)}{[(x^2+1)^2]}$

$\frac{8(x^2+1) - (2x)(8x)}{[(x^2+1)^2]}$

$\frac{[(8)(x^2+1)] - [16x]}{[x^2+1]^4}$

$f'(x)=0$ ← Critical ← Stationary

$f'(x)$ undefine

$5'(x)=0$

$[8x^2+8][x^2+1-4x]=0$

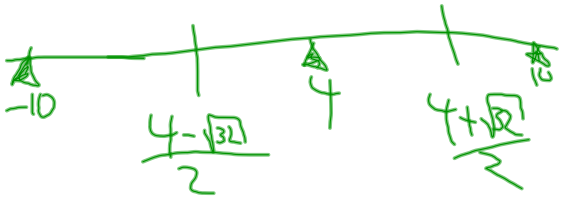
$8x^2+8=0$ (no real solution)

$x^2-4x+1=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{+4 \pm \sqrt{16-(4)(1)(1)}}{2(1)}$

$x = \frac{+4 \pm \sqrt{32}}{2}$



12)

a. $x^{4/3} - 6x^{1/3}$

$y' = 4/3 x^{1/3} - 2x^{-2/3}$

$y' = 4/3 x^{1/3} - \frac{2}{x^{2/3}}$

$y' = \frac{4/3 x^{1/3}(x^{2/3})}{x^{2/3}} - \frac{2}{x^{2/3}}$

$y' = \frac{(4/3 x^{1/3}(x^{2/3})) - 2}{x^{2/3}}$

$y' = \frac{4/3 x - 2}{x^{2/3}}$

$y' = 0 \Rightarrow 3/2$

$4/3 x - 2 = 0$

$4/3 x = 2(3/4)$

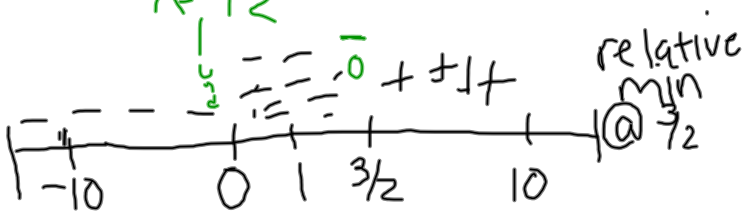
$x = 3/2$

y' under.

$x^{4/3} = 0$

$0^{3/2} = x$

$x = 0$ is under.



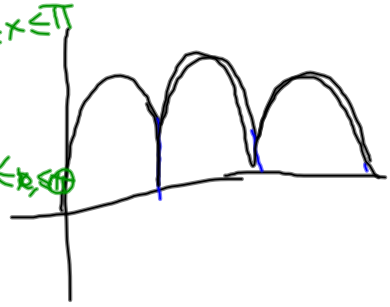
decreasing $(-\infty, 3/2]$

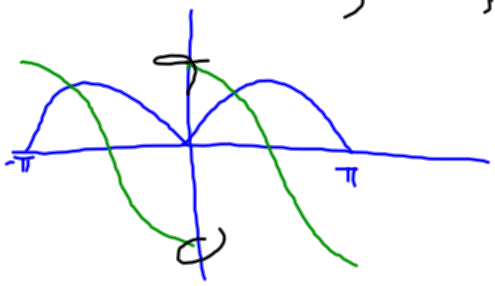
increasing $[3/2, +\infty)$

12b) $y = |\sin x| = \begin{cases} \sin x, & 0 \le x \le \pi \\ -\sin x, & -\pi \le x \le 0 \end{cases}$

$f'(x) = |\sin x|' = \cos x$, $0 \le x \le \pi$ or $-\cos x$, $-\pi \le x \le 0$

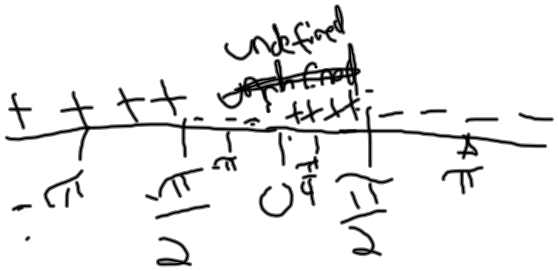
$\sin x$ $-\sin x$





$\cos x = 0$ $-\cos x = 0$

$x = \frac{\pi}{2}$ $x = -\frac{\pi}{2}$



$f(x) = \frac{x^2}{x^2+1}$

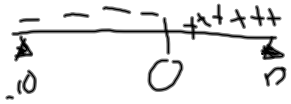
$f'(x) = \frac{(x^2+1)(2x) - (x^2)(2x)}{(x^2+1)^2}$

$= \frac{2x^3 + 2x - 2x^3}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$

always pos

$f(x)=0$ if $2x=0$, $x=0$

$f(x)$ undefine if $(x^2+1)^2=0$, $x^2+1=0$, $x^2=-1$, $x \neq i$



in $[0,\infty)$

de $(-\infty,0]$

min @ $x=0$

$\frac{2x}{(x^2+1)^2}$

$\frac{(2x)'(x^2+1)^2 - 2x(x^2+1)^2}{[(x^2+1)^2]^2}$

$2(x^2+1)^2 - 2x(2(x^2+1)(2x))$

$\frac{[2(x^2+1)][(x^2+1) - 4x^2]}{(x^2+1)^4}$

$[1-3x^2] = 0$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}}$

$f(x)=0$ $\quad$ $f(x)$ undefined

$[2(x^2+1)][(x^2+1) - 4x^2] = 0$

5.2 homework